Dimensioning hospital wards using the Erlang loss model

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OR 50 Conference York - (9-11 sept)

Programma

- **Dimensioning Hospital Wards Using the Erlang Loss Model**

  Presentatie gegeven tijdens OR50 Conference York (9-11 Sept)

- **OR50 Conference "Health Stream" in Vogelvlucht**

Personal background

- **2003 - Graduated at Technical University Delft**
  - Bachelor: Aeronautical Engineering
  - Master: Industrial Organisation
  - Thesis: Modelling and Simulation for Analysis of Operative Care Chains
- **Since 2003**

  Management advisor at VU university medical center (VUmc), division IV
- **Since 2004**

  Researcher (PhD) Faculty of Sciences, VU university Amsterdam (1 day/week)
  Subject: Patient flow and resource allocation in hospitals
- **June 2007**

  Co-founder of PICA, Patient Flow Improvement Center Amsterdam
  www.vumc.nl/pica

VU university medical center

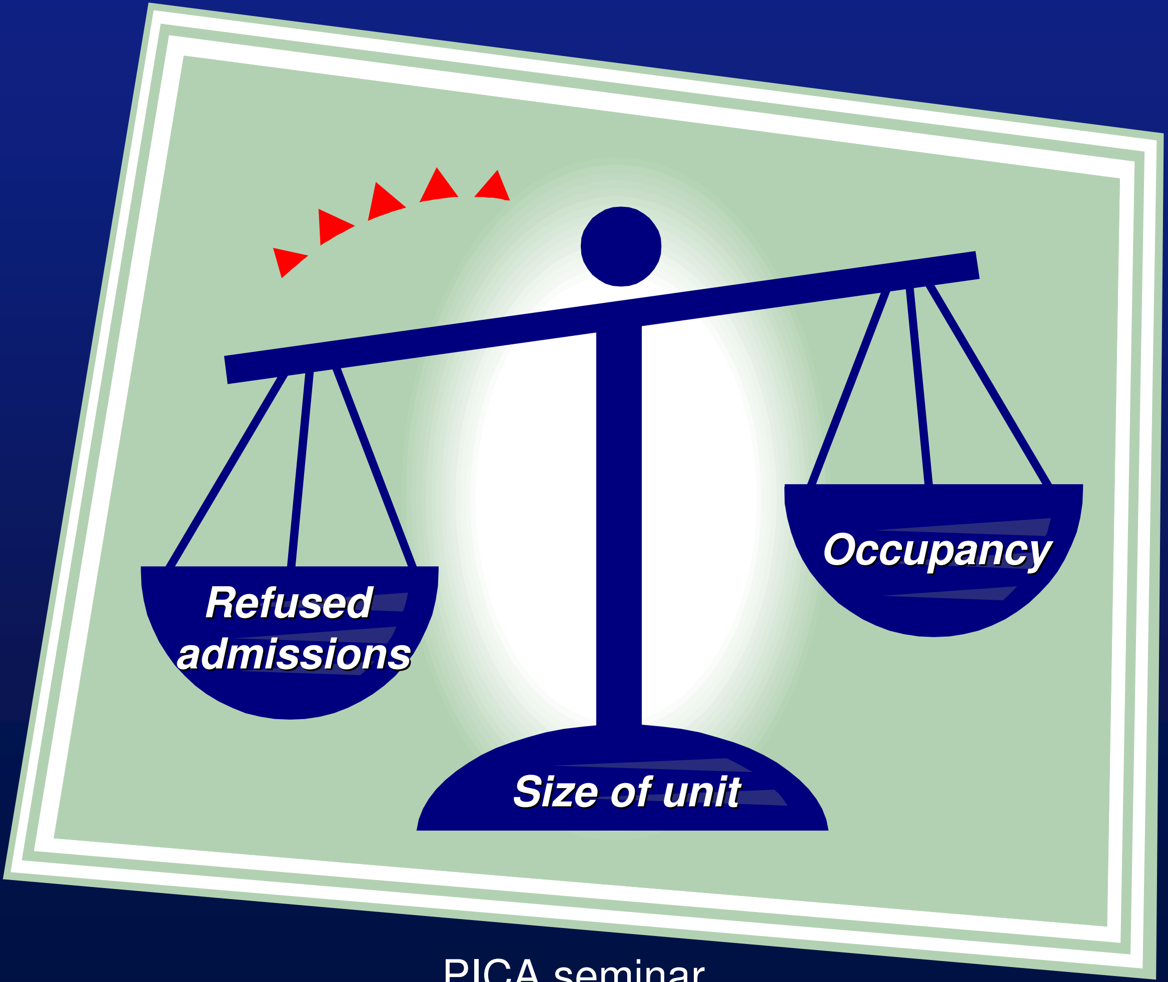
- One of 8 university medical centers
- Factsheet VUmc (2007)

Number of certified beds	713
Admissions	26,169
Day treatments	18,738
Outpatient visits	308,400
ED Admissions	37,477
Average Length of Stay	6.3 days
Personnel (fte)	5,090
Budget	€ 565 million

Relevance

- An increasing number of refused admissions
- Great range in occupancy rates between clinical wards
  → Unbalanced distribution of hospital beds
- Long term strategic studies show that in 5-10 years maximum available bedcapacity is reached
- The discussion about number of beds assigned lacks a rational or quantitative basis

Finding a balance between…



Goals of the study

1. Obtaining insight in key characteristics of in-patient flow by analyzing: admissions, LOS and occupancy rates
2. Demonstrating that in-patient flow can be described by a standard queuing model
3. Quantifying the non-linear relation between the size of a unit, the probability of a refused admission and the occupancy rate.
4. Developing a decision support system for hospital management to evaluate current size of hospital units and to quantify the impact of bed reallocations and merging of wards.

Definitions & terminology

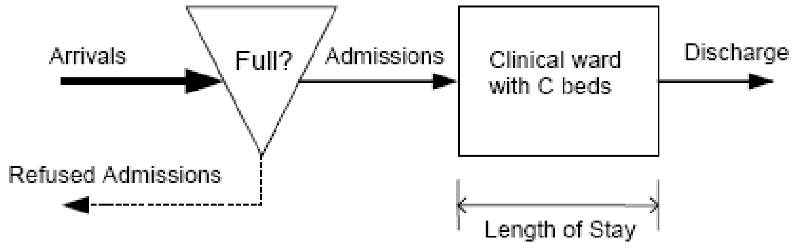


Figure 1. Structural model of patient flow through a clinical ward

Some remarks

- Arrivals = Admissions + Refused Admissions
- In practice, a refused admission can result in a diversion to another hospital or an admission to a non-preferable clinical ward.
- Empirical data on number of refused admissions hard to obtain
- Length of Stay is affected by congestion and delay in the care chain

Data analysis

- Source: Computerized records of admissions to 24 wards over the years 2004-2006
- ED, First Cardiac Aid (FCA), short stay unit (SSU) are excluded
- Wards included in this study:

Category	Ward Description	Operational beds (2004)	Operational beds (2005)	Operational beds (2006)
Intensive Care	Coronary Care Unit (CCU)	6	6	6
Intensive Care	Intensive Care Unit surgical	14	14	14
Intensive Care	Intensive Care Unit medical	12	13	14
Intensive Care	Pediatric Intensive Care Unit (PICU)	9	9	9
Intensive Care	Neonatal Intensive Care Unit (NICU)	20	19	15
Medium Care	Medium Care	7	7	9
Medium Care	Special Care cardiac surgery	6	6	6
Normal Care	NC Cardiac surgery and cardiology	28	28	28
Normal Care	NC Gynaecology	37	37	37
Normal Care	NC Hematology	21	21	21
Normal Care	NC Surgical oncology	27	27	27
Normal Care	NC Internal medicine unit 1	20	20	20
Normal Care	NC Internal medicine unit 2	20	20	20
Normal Care	NC Pediatric unit 1	26	26	26
Normal Care	NC Pediatric unit 2	23	23	25
Normal Care	NC Otolaryngology (ear/nose/throat)	29	26	25
Normal Care	NC Internal lung	23	23	23
Normal Care	NC Neurosurgery and orthopedic surgery	27	27	30
Normal Care	NC Neurology	31	26	24
Normal Care	NC Obstetrics	42	37	31
Normal Care	NC Internal oncology	26	27	27
Normal Care	NC Ophthalmology	21	15	14
Normal Care	NC Trauma surgery	32	30	33
Normal Care	NC Vascular surgery	21	18	23

Scheduled admissions

- Admission pattern is compared with the Poisson distribution
- Histograms were constructed with the daily number of scheduled admissions on the horizontal axis and the frequency on the vertical axis.
- Variance/mean ratio ranges from 0.658 – 1.759 indicating that number of scheduled arrivals in highly variable (ratio = 1 for a Poisson random variable)

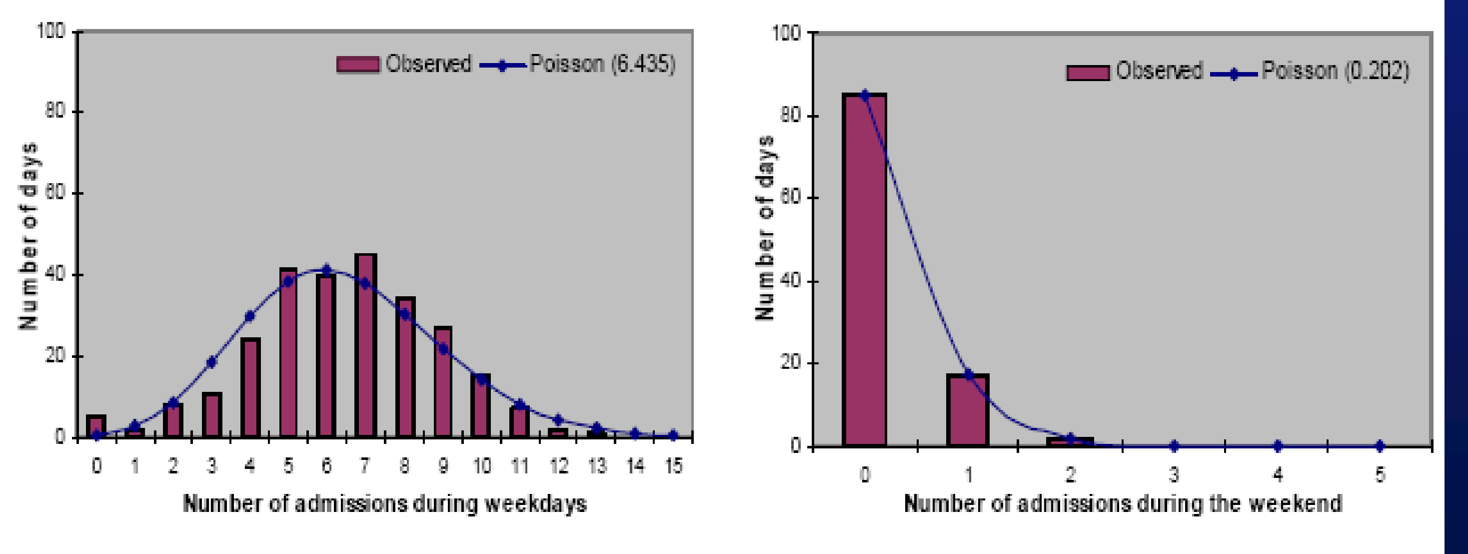


Figure 3. Distribution of scheduled admissions split in weekdays and weekend for hematology (2004)

Unscheduled admissions

- In studies of unscheduled admissions the assumption of a Poisson arrival process has been shown to be realistic [Young JP (1965)]
- Statistical test show that the Poisson distribution provides a good fit for nearly all wards

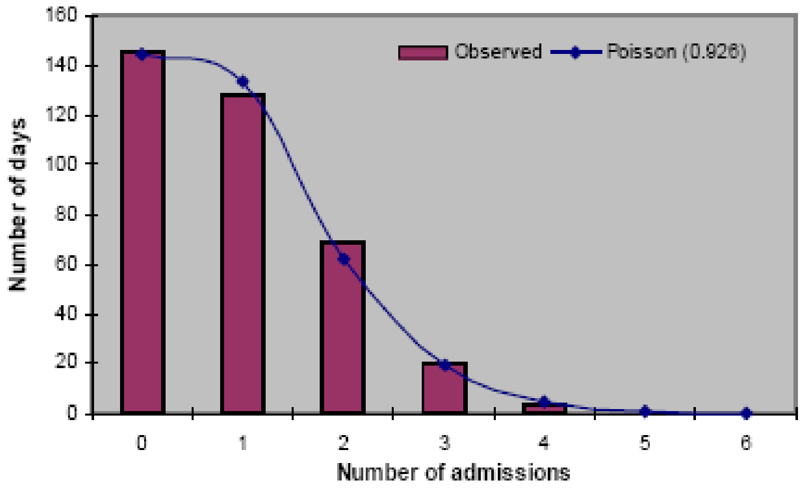
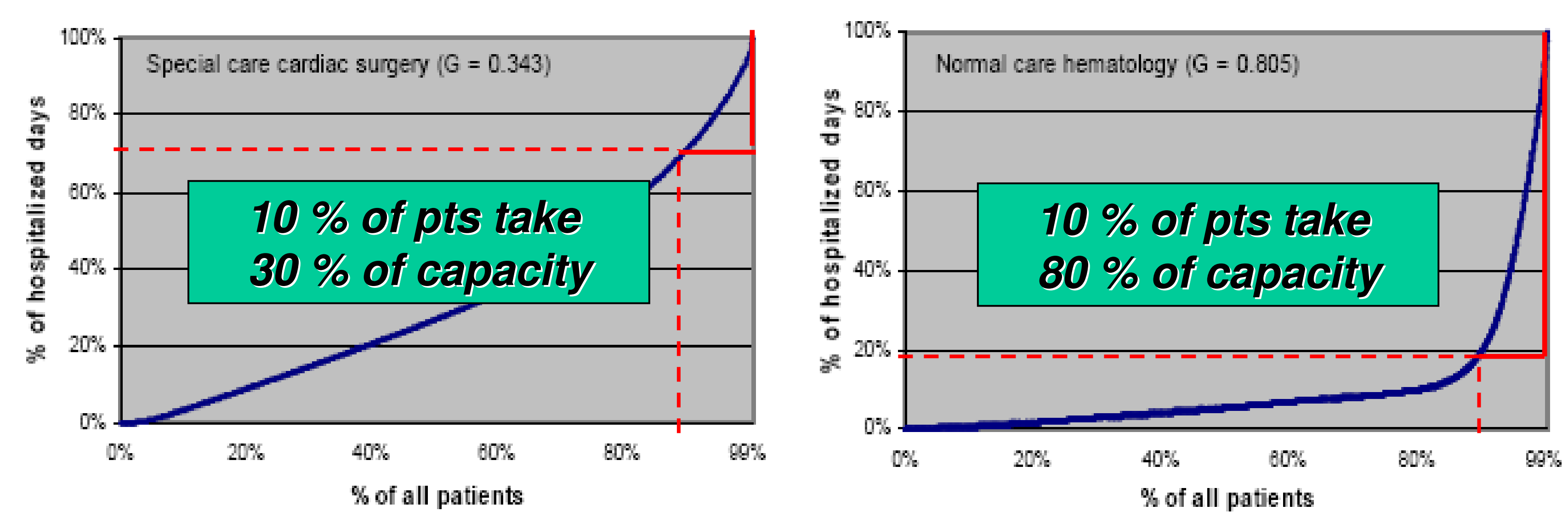


Figure 4. Distribution of unscheduled admissions for the ICU medical (2005)

Length of Stay

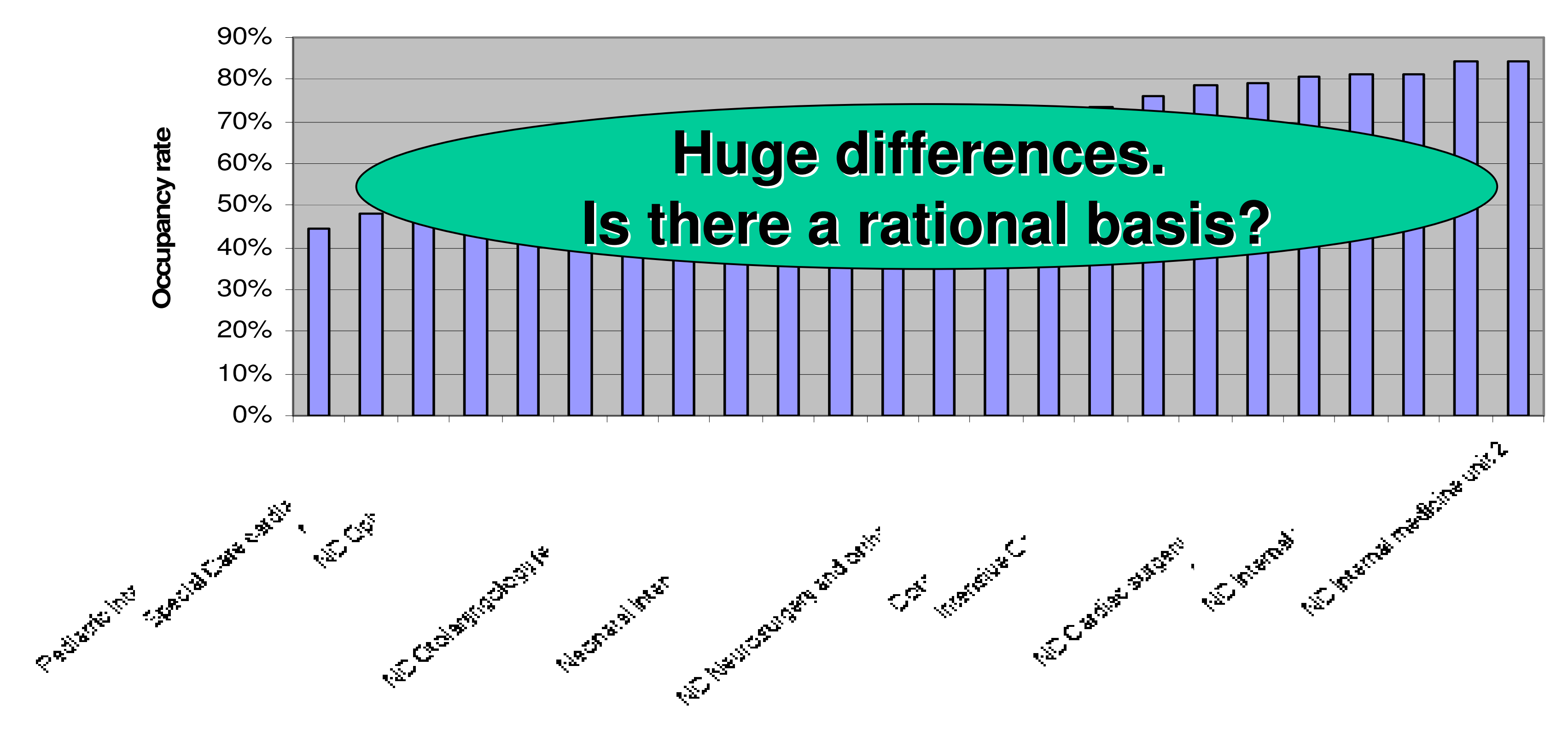
- ALOS = 4 days, ranging from 1.5 days (Obstetrics) to 7.8 days (neonatal ICU)
- $C_v = \sigma / \mu$ ranges from 0.8 (special care cardiac surgery) to 2.7 (hematology)
- LOS distribution was characterized by Lorenz curves



Occupancy rate

$$Occupancy = \frac{Average\ number\ of\ occupied\ beds}{Number\ of\ operational\ beds} = \frac{Admissions\ (per\ time\ unit) \times ALOS\ (time\ unit)}{Number\ of\ operational\ beds}$$

- Occupancy rate ranges from 44% (PICU) to 85% (NC internal medicine)



Discussion data analysis

- Arrivals: Poisson distribution is very reasonable simplifying assumption
- Time-dependent arrival pattern is neglected, because
  - Not that important for strategic and tactical decision making concerning number of beds required
  - Simplicity of use and implementation of final model

- **We believe that:**

"***All models are wrong, but some are useful***"

(George Box, statistician)

Mathematical model

- Queuing approach: Erlang loss model (or M/G/c/c)
- Patients arrive according to a Poisson proces with parameter **λ**
- The LOS is independent and identically distributed with expectation **μ**
- Number of operational beds equals **c**
- Loss fraction

$$P_c = \frac{(\lambda\mu)^c / c!}{\sum_{k=0}^{c} (\lambda\mu)^k / k!} \qquad (1)$$

- Occupancy rate

$$Occupancy \quad rate = \frac{(1 - P_c)\lambda \cdot \mu}{c} \qquad (2)$$

Approximating number of arrivals

- Number of admissions are known from hospital information system
- Numerator in ***(2)*** equals the average number of occupied beds:

$$\text{Average number of occupied beds} = \lambda \cdot \mu\,(1-P_c)$$

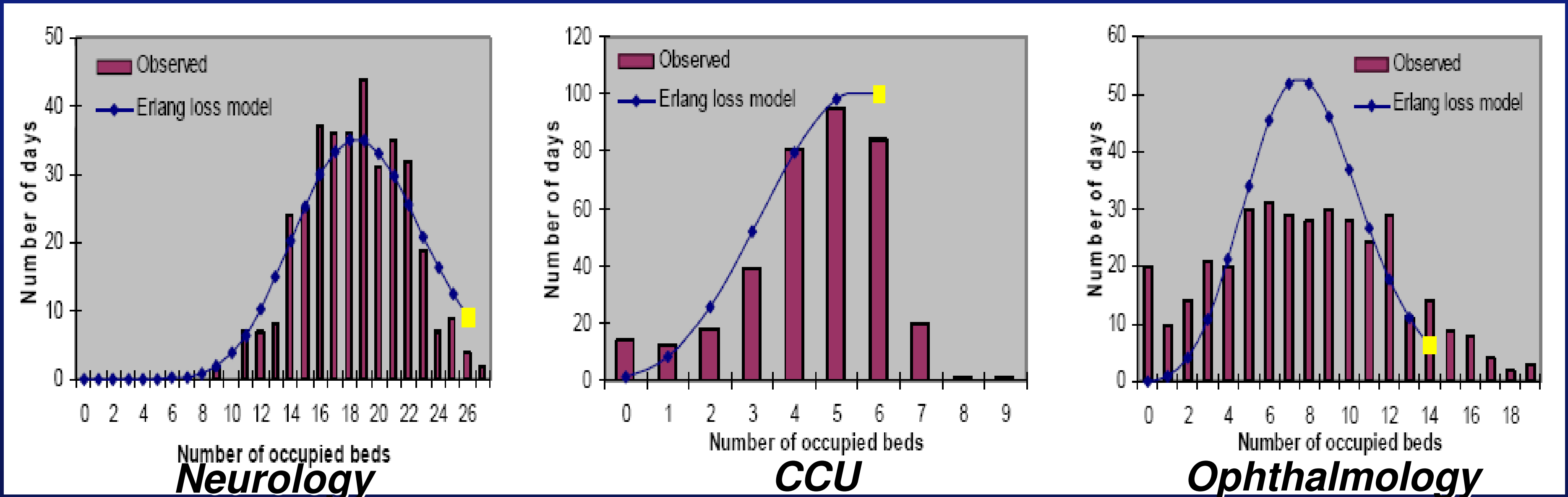
- After substitution of P_c using ***(1)***

$$\text{Average number of occupied beds} = \lambda \cdot \mu \left(1 - \frac{(\lambda\mu)^c / c!}{\sum_{k=0}^{c} (\lambda\mu)^k / k!} \right)$$

- ***λ*** is only parameter left unknown, can be solved numerically

Validation

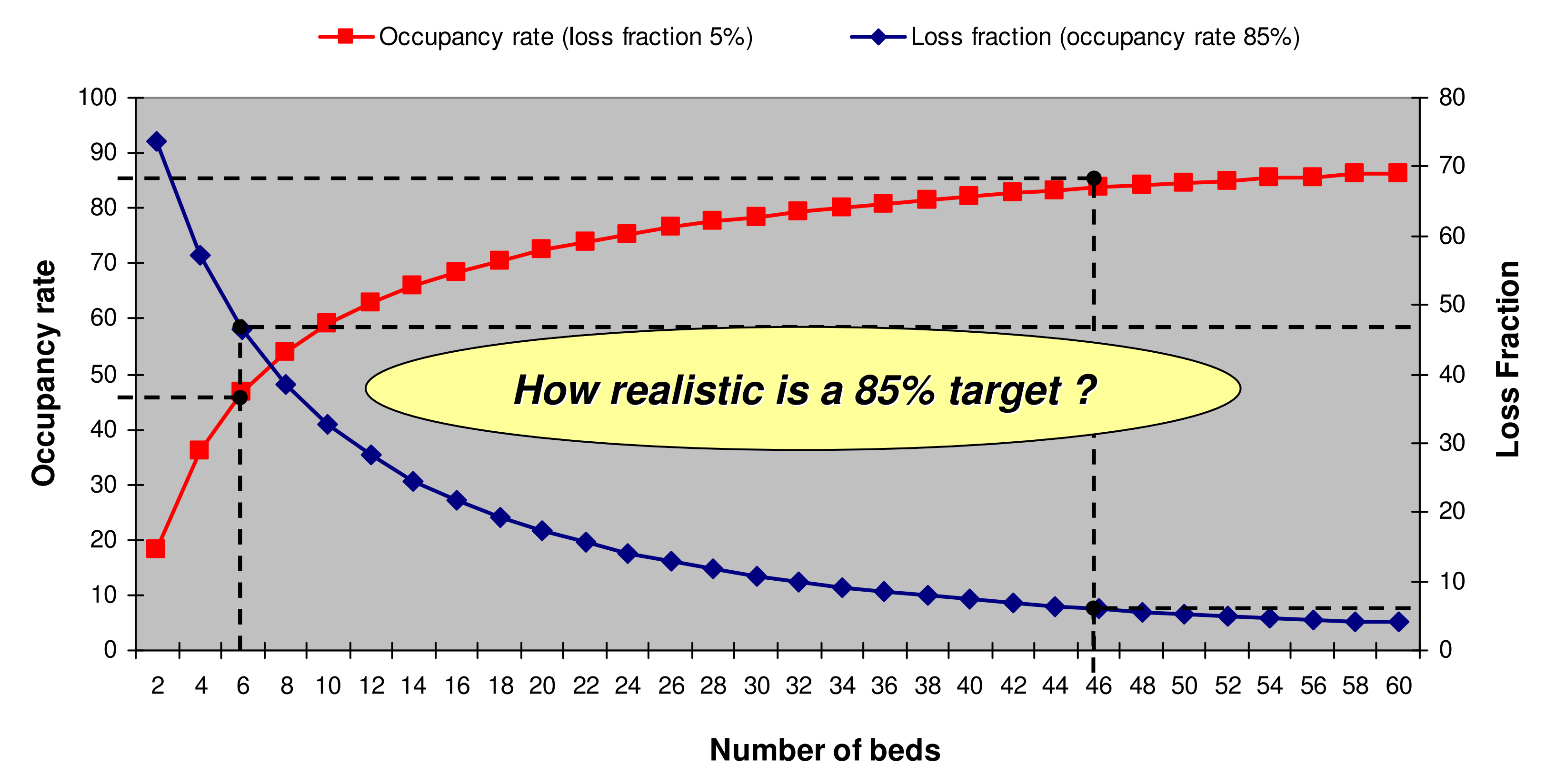
- The observed number of occupied beds was compared with Erlang loss model (curtailed at number of beds c)



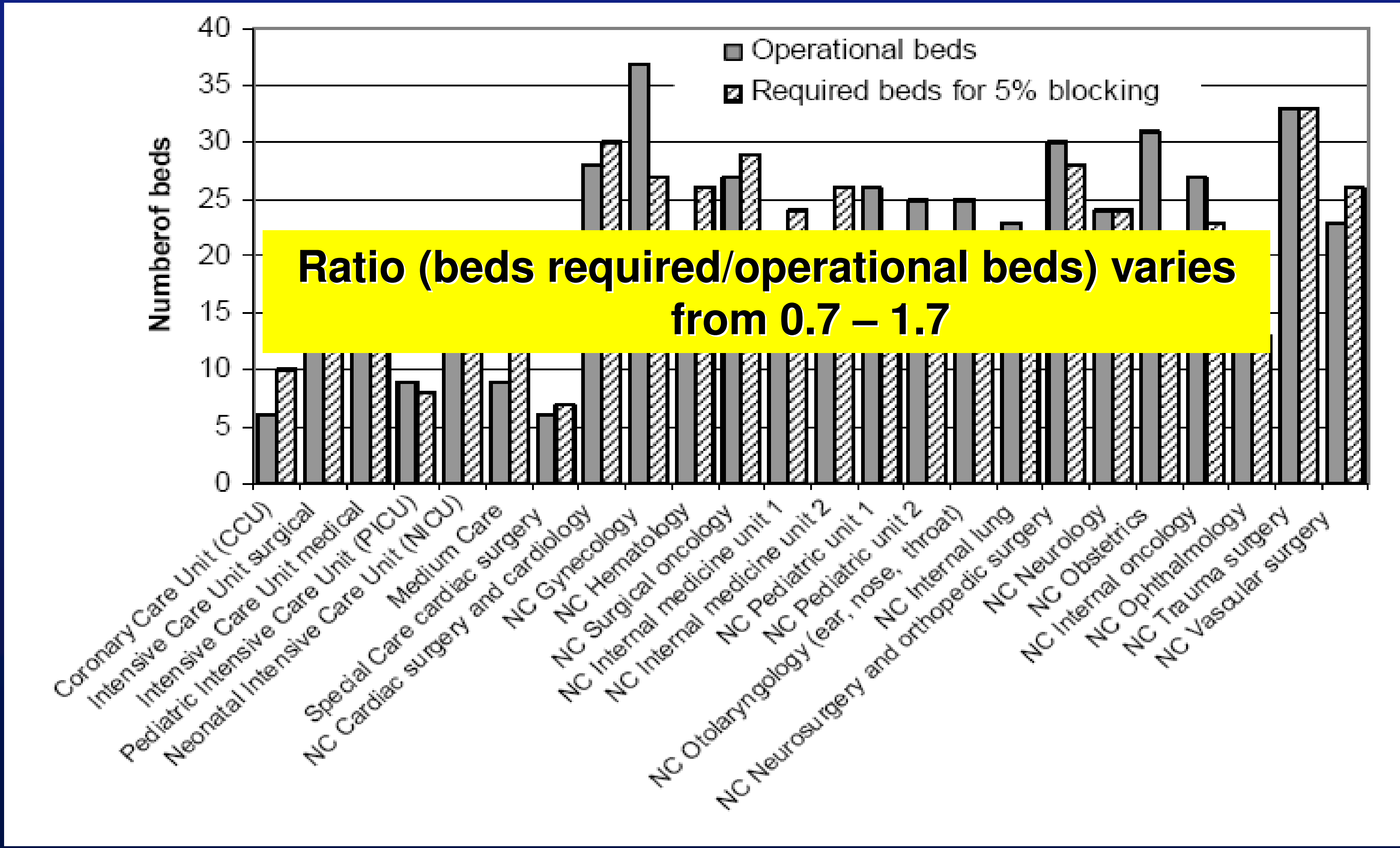
- Goodness-of-fit test was performed
- Model describes number of beds very well, especially for those wards with high fraction of unscheduled admissions

Most important lesson learned

Relation size of a unit, occupancy rate and loss fraction



Evaluation of current size clinical wards



Business case of merging wards

Bed pooling

- Assume we merge (or mix patientflows) the CCU, MC and SC

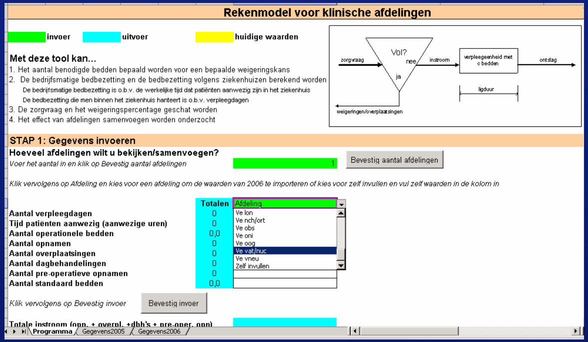
	Parameters (2006)	CCU	MC	SC	
Before merging	Operational beds	6	9	6	
	ALOS [days]	1.694	2.374	1.715	
	Occupancy rate	73.3%	69.8%	48.2%	
	Fraction of refused admissions*	26.2%	13.5%	5.61%	
	Number of beds required for 5% blocking*	10 +	12 +	7	= 29
	After merging CCU, MC, and SC				
After merging	ALOS (weighted)		1.96		
	Number of beds required for 5% blocking*		***22***		
	Occupancy rate		**71.7%**		

*Calculated with the Erlang loss model

Table 7. The effect of merging departments on operational efficiency: case study

Software

- Decision support system in MS Excel



- Online tools: http://www.vumc.nl/afdelingen/pica/Software/

Conclusions

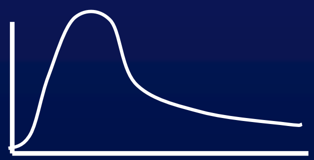
- In-patient flow is characterized by huge variability, even for elective (or scheduled) patient flow
- Historically obtained rights are an important factor
- In-patient flow can be well described by a standard queuing model → gives much insight and is of great value in discussion
- Efficiency can be increased by merging departments (bed pooling) or mixing patient flow → Economies of Scale

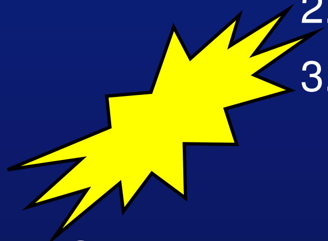
Further research & challenges

- **Process characteristics**

1. Highly variable admission process

2. Highly variable length of stay/ Case duration (OR)/ Treatment time (ED)





- **Organisational structure**

1. Very fragmented due to specialization
2. Many wards that are relatively small
3. Many different queues/slots for e.g.:
   - Out-patient clinics
   - Radiology (MRI / CT)
   - Operating rooms
4. Development of clinical pathways

OR50 Conference York (9-11 sept.)
In vogelvlucht

Wat is Operational Research?

Operational Research (OR) is defined by Morse and Kimball [1951] to be "**a scientific method for providing executive departments with a quantitative basis for decisions regarding operations under their control**", with the added proviso that,

"**quantitative aspects are not the whole story in most executive decisions**".

This umbrella concept of operational research covers many analytic approaches and methods, such as simulation modelling, mathematical programming, decision analysis, cost effectiveness analysis, development of indicators, and methods for forecasting, monitoring and evaluation.

Wat me opviel in York

- Ongeveer 400 deelnemers
  - Veruit de meerderheid ‘Academics’
  - Weinig ‘Practitioners’
- 38 streams, waaronder:
  - Agriculture, Asset Management, Criminal Justice, Cutting & Packing, Energy, Finance, Forecasting, **Healthcare**, Information Systems, Logistics, Maintenance & Reliability, OR & Strategy, OR in Sport, Scheduling, Supply Chain, Systems Think .... and much more....
- Goed gevulde Healthcare stream (**± 20 presentaties**)

Cross-fertilisation between road traffic theory and health OR

Monitoring and reporting surgical wound infections

Een greep uit de Healthcare stream:

Can Lean and Simulation work together in health care?

Radiotherapy scheduling for cancer treatments using metaheuristic

Modelling Catchment Areas for Acute Providers

Analysing and supporting decisions in health: comparison and evaluation of alternative analytical decision models

Validating health economic models: An illustration of external validation of a coronary heart disease model for statins

Modelling trauma hip fracture hospital activities

Modelling costs and effectiveness of treating hepatitis C for injecting drug users in New Zealand

Modelling workforce change in unscheduled care

Development of a Robust Reactive Scheduling System for ICU's

Bedankt voor uw aandacht

www.vumc.nl/pica