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Optimal outpatient appointment scheduling

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Problem context

- Hospital / medical practice
- Elective patients to plan
- One doctor

Objective

- “Optimize” appointment schedule
  - Weighted sum of

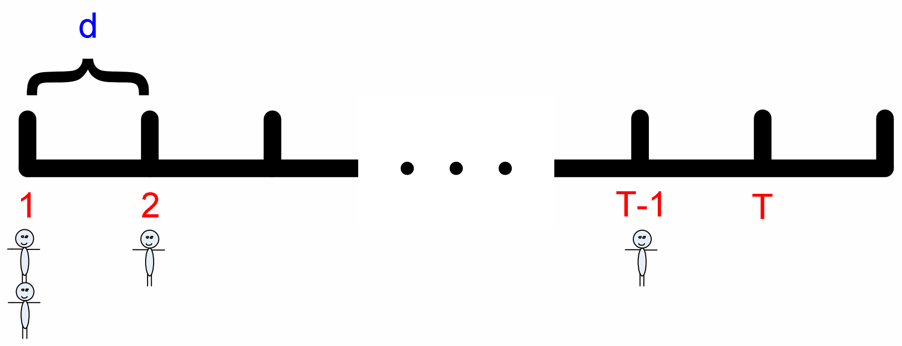
- Mean idle time doctor

- Mean waiting time patients

- Mean tardiness

  - Doctors average overtime given that there is overtime

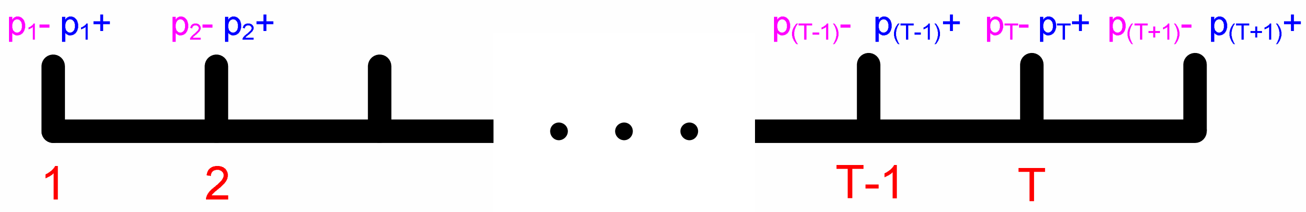
Model



- Variables:
  - T: # intervals
  - d: length interval
  - n: total # patients
  - 1/μ: average service time
- Decision variables:
  - x_t: # of patients scheduled at time t

Model

- Assumptions:
  - Service time ~exp(μ) and i.i.d.
  - Patients right on time



$$p_{t^-}(i) = \mathbb{P}(\text{i patients in queue just before the arrival(s) at interval t})$$

$$p_{t^+}(i) = \mathbb{P}(\text{i patients in queue just after the arrival(s) at interval t})$$

$$\begin{aligned} p_{1^-}(0) &\equiv 1 \\ p_{t^+}(j) &= p_{t^-}(j - x_t) \\ p_{(t+1)^-}(j) &= \sum_{i=j}^{n} p_{t^+}(i) \cdot \mathbb{P}(i - j \text{ departures}) \end{aligned}$$

Mean waiting time of a patient

- WT if k in queue: $k\frac{1}{\mu}$

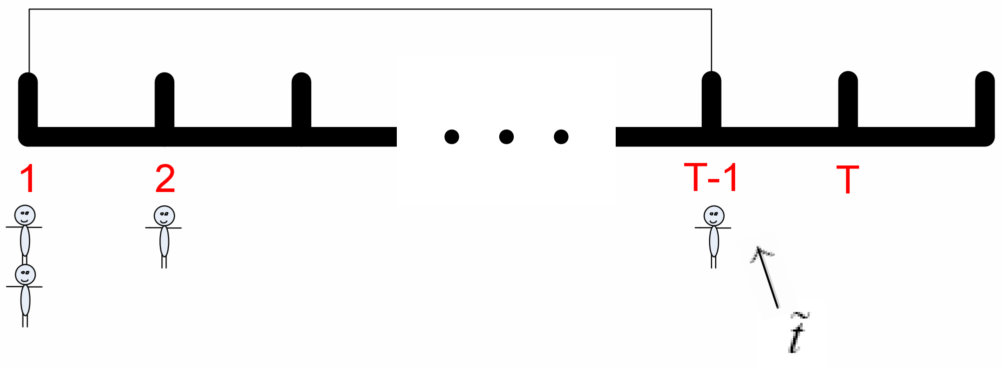
    - Integrate over all patients:

- Mean WT: $W(x) = \frac{1}{N}\sum_{t=1}^{T}\sum_{i=1}^{x_t}\sum_{j=0}^{N} p_{t^-}(j) \cdot (j+i-1)\frac{1}{\mu}$

(x : the schedule)

Mean idle time of a doctor

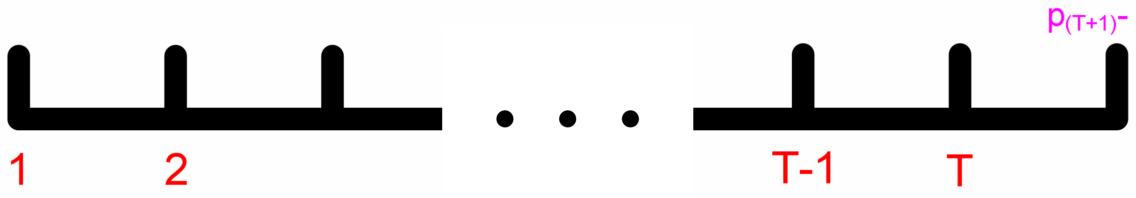
- $I(x)$ (Idle time) $= M(x)$ (Make span) $- \dfrac{N}{\mu}$



- Idle time: $I(x) = \left((\tilde{t} - 1)d + \sum_{j=1}^{N} p_{\tilde{t}+}(j) \cdot \dfrac{j}{\mu} \right) - \dfrac{N}{\mu}$

Mean tardiness of a day

- Tardiness: $$L(x) = \sum_{j=1}^{N} p_{(T+1)^-}(j)\frac{j}{\mu}$$



Including no-shows

- Patients have a change ρ of not coming (i.i.d.)
  - Arrivals: Binomial(x_t,1-ρ)

Objective

- Objective find schedule x that minimize weighted sum of:
  - Waiting time $W(x)$
  - Idle time $I(x)$
  - Taddiness $L(x)$

Local search method

- Neighborhood choice:
  - Use of multimodularity

    (generalization of convexity to lattices)

Neighborhood

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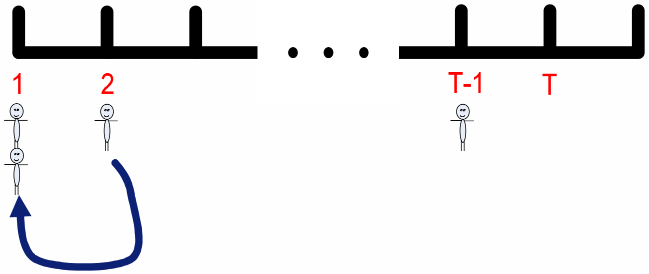
- All permutation of these vectors:

$$\mathcal{V}^* = \left\{ \begin{array}{c} u_1, \\ u_2, \\ u_3, \\ \vdots \\ u_{T-1}, \\ u_T \end{array} \right\} = \left\{ \begin{array}{c} (-1, 0, \ldots, 0, 1), \\ (1, -1, 0, \ldots, 0), \\ (0, 1, -1, 0, \ldots, 0), \\ \vdots \\ (0, \ldots, 1, -1, 0), \\ (0, \ldots, 0, 1, -1) \end{array} \right\}$$

  - all combination of 1-interval shifts starting from *x*

$$x + v_1 + \cdots + v_k \text{ with } \{v_1, \ldots, v_k \subset \mathcal{V}^*\}$$



Internet tool

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Optimal outpatient appointment scheduling tool

Average service time	25	minutes
Number of intervals	10	
Length of interval	30	minutes
Total number of arrivals	10	
Percentage no-shows	5	%
alpha waiting	3	
alpha idle time	1	
alpha tardiness	1	

Press the button to

compute the solutions

Interval	Time	☑ Number of patients of your choice (calc time: several seconds)	☑ ○ Small Neighborhood (Suboptimal, calc time: several minutes) ⦿ Full Neighborhood (Optimal, calc time: several hours)
1	0:00	1	2
2	0:30	1	1
3	1:00	1	1
4	1:30	1	1
5	2:00	1	1
6	2:30	1	1
7	3:00	1	1
8	3:30	1	2
9	4:00	1	0
10	4:30	1	0
Waiting time		16.96 minutes	25.38 minutes
Idle time		82.28 minutes	48.47 minutes
Tardiness		27.55	16.29
Fraction of excess		56.39 %	31.98 %
Makespan		319.78 minutes	285.97 minutes
Lateness		19.78 minutes	-14.03 minutes
Object Value		160.7	140.88

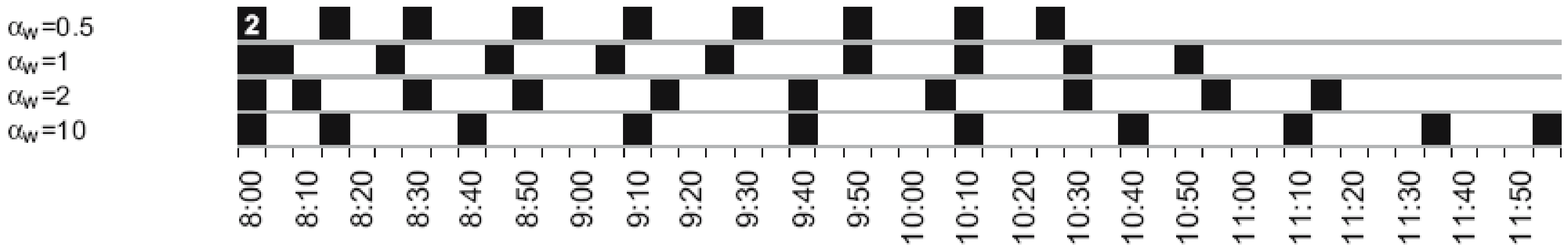
Numeric example

- Inputs Base-Case scenario:

# intervals (T)	48
Length interval (d)	5
# patients (N)	10
Average service time ($1/\mu=\beta$)	20
% no-shows (ρ)	10%

Base-case with α_I=0.2 and α_L=1

- Optimal schedules:



Results

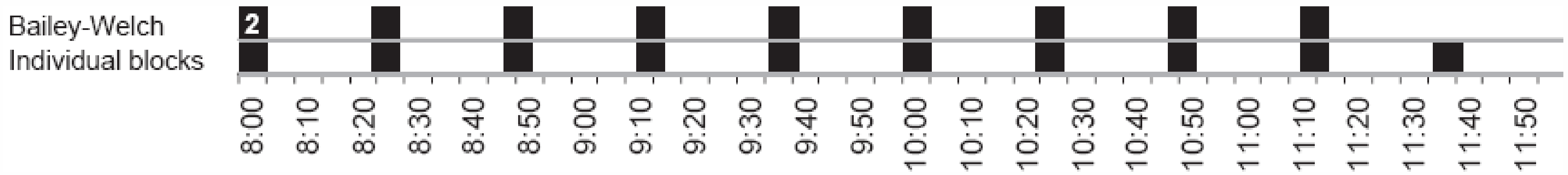


Table 1 Outcome values for different schedules

	$\alpha_W = 0.5$	$\alpha_W = 1$	$\alpha_W = 2$	$\alpha_W = 10$	Individual	Bailey–Welch
Mean waiting time	26.46	19.90	15.35	9.85	12.37	16.75
Mean idle time	21.86	36.69	54.02	88.58	72.14	50.07
Mean tardiness	7.99	9.60	12.61	29.79	19.62	11.42
Object value ($\alpha_W = 0.5$)	25.59				40.23	29.81
Object value ($\alpha_W = 1$)		36.83			46.41	38.18
Object value ($\alpha_W = 2$)			54.12		58.78	54.94
Object value ($\alpha_W = 10$)				146.00	157.72	188.95

Optimal schedules

- α_W=2, α_I=0.2 and α_L=1 & same workload as basecase
- No-shows against service time

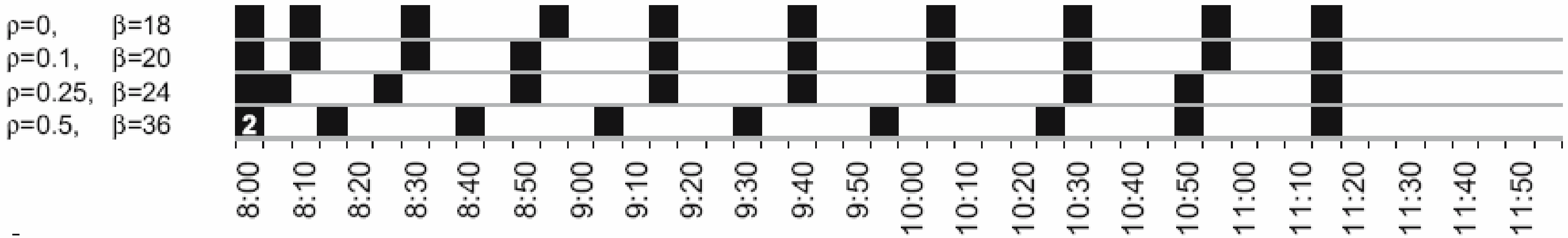


Table 2 Outcome values(ρ against β)

	$\rho = 0, \beta = 18$	$\rho = 0.1, \beta = 20$	$\rho = 0.25, \beta = 24$	$\rho = 0.5, \beta = 36$
Mean waiting time	13.43	15.35	18.93	27.29
Mean idle time	51.67	54.02	56.96	60.66
Mean tardiness	10.04	12.61	17.28	28.59
Object value	47.24	54.12	66.53	95.29

Optimal schedules

- α_W=2, α_I=0.2 and α_L=1 & same workload as basecase
- Number of patients against service time

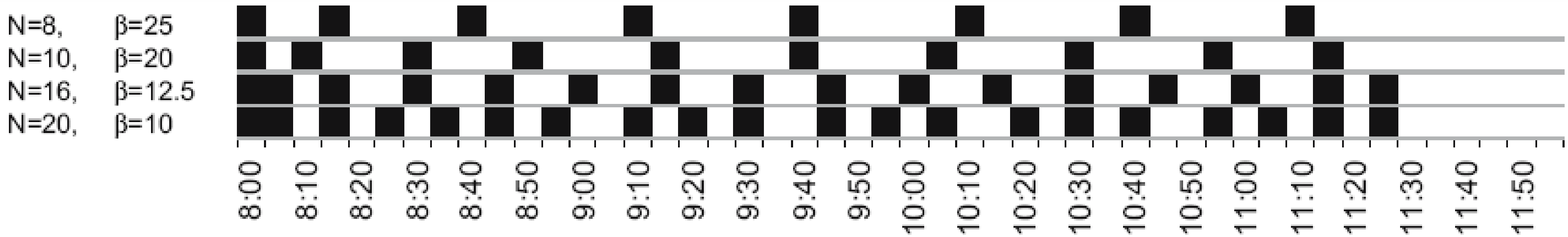


Table 3 Outcome values(N against β)

	$N = 8, \beta = 25$	$N = 10, \beta = 20$	$N = 16, \beta = 12.5$	$N = 20, \beta = 10$
Mean waiting time	16.74	15.35	11.83	11.09
Mean idle time	54.82	54.02	53.53	49.30
Mean tardiness	15.56	12.61	8.10	5.60
Object value	60.00	54.12	42.47	37.63

Optimal schedules

- α_W=2, α_I=0.2 and α_L=1 & same workload as basecase
- Number of patients against no-shows

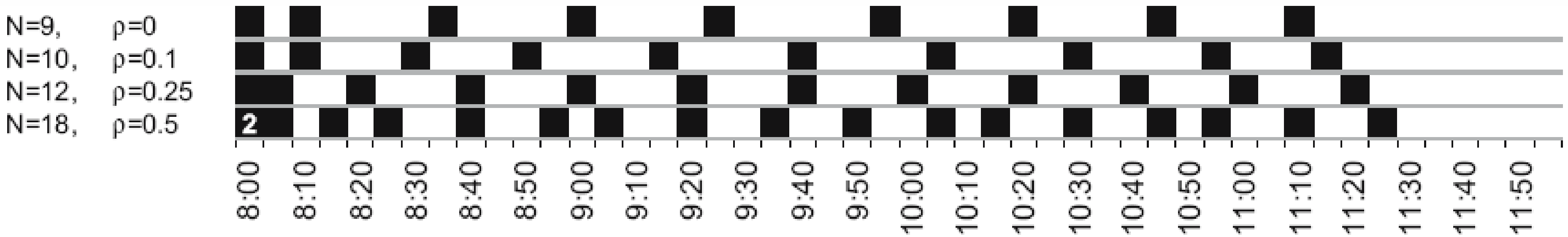


Table 4 Outcome values(N against ρ)

	$N = 9, \rho = 0$	$N = 10, \rho = 0.1$	$N = 12, \rho = 0.25$	$N = 18, \rho = 0.5$
Mean waiting time	14.44	15.35	17.48	21.73
Mean idle time	50.12	54.02	56.43	58.07
Mean tardiness	10.83	12.61	14.63	17.35
Object value	49.73	54.12	60.89	72.43

Limitations

- Assumptions:
  - Exponential distributed service times
  - Patients always right on time

- But:
  - Multimodularity proof <u>don't make any</u> assumption about service times!
    - So local search method can always be used!

Further research…

- Including emergency patients (negative no-shows)
- Including earliness/ lateness patients
- Erlang distributed service times

The end

Questions???